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BOOK REVIEWS

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Books

R. P. AGARWAL AND P. J. Y. WONG, *Error Inequalities in Polynomial Interpolation and Their Applications*, Mathematics and Its Applications **262**, Kluwer Academic, Dordrecht, 1993, x + 365 pp.

A main theme of this book, which lies behind the selection and organization of the material in it, is the use of interpolation in the theory of ordinary differential equations. Inasmuch as different initial and/or boundary conditions for a differential equation lead naturally to various interpolation techniques to approximate the solution, the book discusses a number of different methods for constructing interpolants. Each of the six chapters features an interpolation method or several related methods and also contains extensive discussion of error estimates and applications. Most applications are to the theory of differential equations, relating to such questions as existence and uniqueness of solutions.

The first four chapters of the book deal with various schemes for constructing algebraic polynomial interpolants. Chapter 1 treats Lidstone interpolation, which is the construction of a polynomial p of degree at most $2n-1$ with prescribed values of $p^{(2i)}(0)$ and $p^{(2i)}(1)$, for $i=0, ..., n-1$. Chapter 2 deals with generalized Hermite interpolation, in which a polynomial p of degree at most $n-1$ is constructed, using a set of r nodes, with $n \geqslant r$; at each node the value of p itself and the values of some number of its consecutive derivatives are prescribed, with the total number of conditions equal to n. These conditions include as special cases classical Hermite interpolation ($n=2r$, with values of p and p' given at each point) and classical Lagrange interpolation ($n=r$). Chapter 3 deals with Abel–Goncharov interpolation, that is, the construction of a polynomial p of degree at most n upon points $a_0 \leqslant a_1 \leqslant \cdots \leqslant a_n$ with preassigned values $p^{(i)}(a_i)$, for $i=0, ..., n$. Chapter 4 deals with various "miscellaneous" interpolation problems which can arise from different types of initial and boundary data.

The last two chapters of the book deal with piecewise polynomial approximation, with the last chapter dealing particularly with spline interpolation. Both of these chapters also include some discussion of approximation of functions of two variables.

In all chapters, the error estimates are carefully constructed and contain actual constants, in many cases best possible. Demonstrably useful in the applications to differential equations, almost all of the error estimates are cast in terms of the norm (usually the uniform norm, sometimes an L^p norm) of a high derivative of the function being approximated—essentially, the error incurred in an interpolation of n pieces of data is measured by the norm of the nth derivative of the function being approximated. Considering the detail and precision of these estimates, it is perhaps unfortunate that the authors did not address such error estimates which remain useful and valid when the function approximated is not very smooth. But no book can contain everything.

Some particular positive points in the book are found in unexpected places. For example, Section 5.2 (innocuously called "Preliminaries") contains some very interesting polynomial and function inequalities for derivatives, including a serious discussion of the constant in the Markov inequality in (unweighted) $L^2[a, b]$.

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To sum up, this book was written with particular applications in mind, and it will no doubt find uses among specialists in differential equations. Otherwise, the wealth of detail and the precision of the error estimates in it go beyond what is generally available in book or monograph form and commend the work to a more general audience.

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M. Holschneider, *Wavelets: An Analysis Tool*, Oxford Mathematical Monographs, Clarendon, Oxford, 1995, xiii + 423 pp.

This monograph provides a solid introduction to the theory of wavelets. Unlike many other introductory texts, which put an emphasis on the discrete wavelet transform, the main focus of this book is the continuous wavelet transform, in the first chapter formally defined as

$$W[g, s](b, a) = \int_{-\infty}^{+\infty} \frac{1}{a}\, \bar{g}\left(\frac{t-b}{a}\right) s(t)\, dt.$$

This transforms a function s over the real line to a function $W[g, s]$ over the upper half-plane $\mathbb{H} = \{(b, a) \mid b \in \mathbb{R},\ a > 0\}$. The wavelet g is assumed to be localized in time and henceforth the wavelet coefficients $W[g, s](b, a)$ analyze the function s at position b with scale a. A formula involving another wavelet h reconstructs a function over the real line from a function over the upper half-plane. The formula leads to a stable inversion of the wavelet transform if the pair g, h satisfies a specified condition. This condition reduces to the wavelet admissibility condition in the case where $g = h$. Results of this type are proved where the functions to be analyzed (reconstructed) are taken from L^2 spaces or spaces of distributions. The analyzing (reconstructing) wavelets are required to be elements of L^2 spaces or spaces of highly localized and regular functions, respectively.

In the second chapter, the author studies (partial) reconstruction of functions over the real line from the wavelet transform on subsets of the upper half-plane $\mathbb{H}$. As an application of the Poisson summation formula, a wavelet analysis over the one-dimensional torus is constructed. It is also demonstrated that the reconstruction of a function from its wavelet coefficients on certain grids in the upper half-plane requires the wavelets under consideration to induce Bessel sequences or frames. The subject of Chapter 3 is multiresolution analysis of several L^2 spaces. Compactly supported orthonormal wavelet bases are constructed from Lagrange interpolation spaces, hence proving a well-known result by Daubechies. In the fourth chapter the connection between local regularity and pointwise differentiability of functions and the behavior of its wavelet transform at small scales is discussed. In this manner, the continuous wavelet transform is used as an analysis tool to study the regularity of a typical trajectory of a Brownian motion, the Riemann–Weierstrass function, and certain dynamical systems. In Chapter 5, the wavelet transform over locally compact groups is treated. Apart from its theoretical value, this chapter also puts the results of the first three chapters in a broader perspective. The chapter ends with an interesting example, the inversion of the Radon transform over the two-dimensional plane. The sixth and last chapter of the book introduces Banach spaces that are characterized by the localization of the wavelet coefficients over the half-plane. In this context, Calderòn–Zygmund operators are discussed and the author refers to the books of Meyer and Coifman for further reading.

The book contains a large number of interesting topics which could not all be mentioned in this review. The text is enriched with instructive, nontrivial examples and illustrations. The